Study of the Limit of Transmission Problems in a Thin Layer by the Sum Theory of Linear Operators

We consider a family \( P(d) \), where \( d \) is a small positive parameter, of singular elliptic transmission problems in the juxtaposition \([-1, d] \times G\) of two bodies, the cylindric medium \( V = [-1, 0] \times G \) and the thin layer \( V(d) = [0, d] \times G \). It is assumed that the coefficient in \( V(d) \) is \( 1/d \). Such problems model for instance heat propagation between the body \( V \), the layer \( V(d) \) (when supposed with infinite conductivity) and the ambient space.

In a first step, we perform a rescaling in the thin layer to transform the problem \( P(d) \) in a problem \( Q(d) \) set in the fixed domain \([-1, 1] \times G\). Then we write problem \( Q(d) \) in the form of a sum of linear operators and we show that the sum theory developed by Da Prato-Grisvard works. This gives an explicit writing of the strong solution \( u(d) \) as a Dunford Integral in the \( L^p \) spaces, \( p > 1 \).

In the second step, we study the behavior of \( u(d) \) as \( d \) tends to 0. We deduce that the family of solutions \( u(d) \) converges in \( L^p \) to a function \( u \) in the case of second member in \( L^p \) and converges in \( W^{(1+2s,p)} \) for a second member in \( W^{(2s,p)} \) (s in \( [0, 1/2] \)). Moreover, in virtue of the techniques used in the study of abstract differential equations, we then prove that the restriction of the limit \( u \) to \([-1, 0] \times G\) is in fact, the solution to an elliptic problem on \([-1, 0] \times G\), with a boundary condition of Ventcel’s type and it has an optimal regularity.

Finally, we go back to our first problem, in order to translate all the above results on the solution \( v(d) \) of \( P(d) \).